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QCD corrections to large- $m_t$  electroweak effects in  $\Delta r$ .  
 An effective field theory point of view.<sup>★</sup>

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# ABSTRACT

By using effective field theory techniques for the standard model, we discuss the issue of what  $\mu$  scale is the appropriate one in the QCD corrections to the large- $m_t$  electroweak contributions to  $\Delta r$ . This needs the construction of an effective field theory below the top quark. We argue that, while matching corrections do verify the simple prescription of taking  $\mu \simeq m_t$  in  $\alpha_s(\mu)$ , logarithmic (i.e.  $\sim \log m_t$ ) corrections do not, and require the use of the running  $\alpha_s(\mu)$  in the corresponding renormalization group equation.

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The high precision experiments done at LEP<sup>[1]</sup> and the SLC<sup>[2]</sup> have recently motivated the interest in the  $\alpha_s$  corrections to the large- $m_t$  contributions to observables in the standard model (SM). We would like to concentrate here on  $\Delta r$  for its simplicity.

According to refs. [3-6],  $\Delta r$  can be expressed as<sup>‡</sup>

$$\begin{aligned} \Delta r(top) \approx & -\frac{c^2}{s^2} \frac{3m_t^2 G_F \sqrt{2}}{(4\pi)^2} \left(1 - \frac{\alpha_s(\mu)}{\pi} \frac{6 + 2\pi^2}{9}\right) + \\ & + \frac{g^2}{(4\pi)^2} \frac{1}{2} \left(\frac{c^2}{s^2} - \frac{1}{3}\right) \log\left(\frac{M_W^2}{m_t^2}\right) \left(1 + \frac{\alpha_s(\mu)}{\pi}\right) \quad , \end{aligned} \quad (1)$$

where we have only kept the leading and next-to-leading  $m_t$  dependence.

Along with these results, there has appeared the discussion of the  $\mu$  scale at which one is supposed to evaluate  $\alpha_s(\mu)$  in these expressions<sup>§</sup>, and the parameters in terms of which one ought to express the result, i.e. whether  $\overline{MS}$ , or on-shell, etc. For this, a prescription has been designed that says<sup>[4]</sup> that corrections coming from the  $(t, b)$  doublet should be computed with  $\alpha_s(m_t)$ . This prescription would then say that in all of the above expressions  $\alpha_s(\mu)$  should be taken as  $\alpha_s(m_t)$ . We would like to explain what an effective field theory (EFT) point of view shows about this issue. We shall see that while this prescription works for the power-like terms (those that go like  $m_t^2$ ), the renormalization group (RG) supplies us with a different result for the logarithmic terms (those that go like  $\log m_t$ )<sup>¶</sup>.

The EFT<sup>[7]</sup> is a very suitable framework for gaining control over the problem of the interrelation of scales and quantum corrections just because of the very nature of an EFT. Indeed, in the process of constructing it, one must clearly identify the

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<sup>‡</sup> There is a typographical error in the  $\log M_W^2/m_t^2$  term of  $\Delta r(top)$  in ref. [6], which appears with an overall minus sign with respect to our expression. We thank F. Jegerlehner for confirming this.

<sup>§</sup> In principle three scales appear in these loops:  $m_t, M_Z, m_b$ .

<sup>¶</sup> From an effective field theory point of view these two types of contributions are totally different. While the former (i.e. power-like) come from "matching", the latter (i.e. logarithmic) come from "running". See below.

relevant scales appearing in the problem. The construction goes as follows<sup>[8] [7]</sup>. For a certain physical process occurring at an energy scale  $E_0$ , one starts with the bare Lagrangian (which is valid at an energy  $E \gg E_0$ , where  $E$  is bigger than any particle's mass) and integrates out all particles whose masses are larger than  $E_0$ . In this process, every time one integrates out a particle one does a large mass expansion, so that there appears a tower of higher dimensional operators weighted with the appropriate inverse powers of the particle's mass one has just integrated out. Consequently, one has to include new counterterms to compensate for the different high energy behavior of the effective theory under construction with respect to the original full theory, so that both the effective and the full theory yield the same physics. This is typically done at a scale  $\mu^*$  equal to the particle's mass to minimize the effect of large logarithms. Power corrections appear right at this point. This is called a "matching condition". Below the particle's mass one can then use the powerful machinery of the renormalization group to obtain the effective Lagrangian relevant at lower scales; in other words, one "runs" the Lagrangian down to the lower scales. It is then that one obtains the typical logarithms between two scales. If in the process of doing so one encounters another particle, then one again has to integrate it out and find the matching condition at threshold for the corresponding counterterm, so that the procedure can be iterated until the energy scale  $E_0$  is reached. Only particles lighter than the scale  $\mu$  one is at, at every moment, are to be included in the running, i.e. in the RG equations. Therefore, in an EFT all the contributions are either "matching" or "running", and there is nothing else.

In this paper we shall be concerned with the QCD corrections to the large- $m_t$  one-loop electroweak corrections. Therefore, for all practical purposes, one may think as if the top quark were the heaviest particle in the SM, much heavier than the Higgs boson, which is taken to be nearly degenerate with the W and Z.

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\* For convenience, we shall always work in the  $\overline{MS}$  scheme. Accordingly, couplings and masses will depend on the scale  $\mu$ . Unless otherwise stated, only the pole mass will have no explicit dependence on  $\mu$  in the text.

This automatically kills the  $\log M_H/M_W$  contributions and leaves the  $m_t^2$  and the  $\log m_t/M_W$  ones, which are those we are interested in.

The general philosophy will parallel that so successfully used in the context of grand unified theories<sup>[9]</sup>. There is of course a very important difference, namely that, upon integration of the top quark, the resulting effective theory will no longer exhibit an explicit linear  $SU_2 \times U_1$  invariance. This would make a full account of the corresponding RGE's very cumbersome. Luckily we may keep only those contributions that are strictly relevant.

Let us start with the full SM at  $\mu > m_t$ . At  $\mu = m_t$ , one integrates the top out obtaining<sup>\*</sup>

$$\begin{aligned}
\mathcal{L}_{eff} = & (1 + g^2 \delta Z_+(\mu)) W_\mu^+ \partial^2 W^{-\mu} + \frac{g^2}{4} (v^2 + \delta v_+^2(\mu)) W_\mu^+ W^{-\mu} + \\
& + \frac{1}{2} (1 + g^2 \delta Z_3(\mu) + g^2 \delta Z_{3Y}(\mu)) W_\mu^3 \partial^2 W_3^\mu + \\
& + \frac{1}{2} (1 + g'^2 \delta Z_Y(\mu) + g'^2 \delta Z_{3Y}(\mu)) B_\mu \partial^2 B^\mu + \\
& + \frac{1}{2} (g W_3^\mu - g' B^\mu) \left[ \frac{1}{4} (v^2 + \delta v_3^2(\mu)) - \delta Z_{3Y}(\mu) \partial^2 \right] (g W_\mu^3 - g' B_\mu) + \\
& + \bar{\psi} i \not{D}(g W^+, g W_3, g' B) \psi
\end{aligned} \tag{2}$$

from the diagrams of fig. 1. Here  $\psi$  stands for all the fermions but the top. Notice that we have dealt with  $W_3 - B$  mixing by including a  $\partial^2$  operator in the form of a generalized mass term in eq. (2). This will make the subsequent diagonalization very simple since the neutral mass eigenstate is still of the form  $g W_3 - g' B$ , like at tree level. Certainly, there will also be a tower of higher dimensional operators suppressed by the corresponding inverse powers of the top quark mass, but we shall neglect them throughout. Possible four-fermion operators are irrelevant to the discussion that follows and are also disregarded. We also postpone the study of the  $Z b \bar{b}$  vertex to a future analysis<sup>[10]</sup>.

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\* For instance, in the context of grand unification,  $g$  in the following equation would be the value of the  $SU_2^{EW}$  gauge coupling constant at the scale  $\mu = m_t$  as obtained from the running of the corresponding  $\beta$  function between  $M_{GUT}$  and  $m_t$ .

We can now redefine our fields in order to have standard kinetic terms. This yields

$$\begin{aligned} \mathcal{L} = & W_\mu^+ \partial^2 W^{-\mu} + \frac{g_+^2(\mu)}{4} (v^2 + \delta v_+^2(\mu)) W_\mu^+ W^{-\mu} + \frac{1}{2} W_\mu^3 \partial^2 W^{3\mu} + \frac{1}{2} B_\mu \partial^2 B^\mu + \\ & + \frac{1}{2} (g_3(\mu) W_3^\mu - g'(\mu) B^\mu) \left[ \frac{1}{4} (v^2 + \delta v_3^2(\mu)) - \delta Z_{3Y}(\mu) \partial^2 \right] (g_3(\mu) W_{3\mu} - g'(\mu) B_\mu) \\ & + \bar{\psi} i \not{D} (g_+(\mu) W^+, g_3(\mu) W_3, g'(\mu) B) \psi , \end{aligned} \quad (3)$$

where, to the order we are working, i.e. one loop:

$$\begin{aligned} g_+^2(\mu) &\approx g^2 (1 - g^2 \delta Z_+(\mu)) \\ g_3^2(\mu) &\approx g^2 (1 - g^2 \delta Z_3(\mu) - g^2 \delta Z_{3Y}(\mu)) \\ g'^2(\mu) &\approx g'^2 (1 - g'^2 \delta Z_Y(\mu) - g'^2 \delta Z_{3Y}(\mu)) \end{aligned} \quad (4)$$

Notice that below the top quark mass the initially unique coupling constant  $g$  has split into  $g_+$  and  $g_3$ <sup>[11]</sup>. Similarly  $v_+^2(\mu) \equiv v^2 + \delta v_+^2(\mu)$  and  $v_3^2(\mu) \equiv v^2 + \delta v_3^2(\mu)$  are also different. The matching conditions are very easily obtained since they are nothing else than the diagrams of fig. 1 evaluated at  $\mu = m_t$ . One obtains for instance, at one loop,

$$\begin{aligned} \delta v_+^2(\mu) &= \frac{N_c}{(4\pi)^2} m_t^2 (2 \log(\mu^2/m_t^2) + 1) \quad , \\ \delta v_3^2(\mu) &= \frac{N_c}{(4\pi)^2} m_t^2 2 \log(\mu^2/m_t^2) \quad . \end{aligned} \quad (5)$$

Therefore this means that

$$\delta v_+^2(m_t) = \frac{N_c}{(4\pi)^2} m_t^2 \quad , \quad \delta v_3^2(m_t) = 0 \quad . \quad (6)$$

Analogously,

$$\delta Z_{3Y}(m_t) = 0 \quad , \quad g_+(m_t) = g_3(m_t) = g \quad \text{and} \quad g'(m_t) = g' \quad . \quad (7)$$

Equation (7) says that the coupling constants are continuous across the threshold. This is true as long as one keeps only the leading logarithms. In general there

are non-logarithmic pieces that modify (7) such as, for instance, the non-log term in the first of eqs. (5). The point is that this term in (5) is multiplied by  $m_t^2$  (i.e. a non-decoupling effect) and therefore contributes (in fact dominates) for large  $m_t$ , whereas the same does not happen in (7). Therefore, non-log corrections to (7) do not affect the large- $m_t$  discussion that follows.

In order to obtain the effective Lagrangian at the relevant lower scales  $\mu \simeq M \equiv M_W, M_Z$ <sup>★</sup> one has to scale this Lagrangian down using the RGE for each "coupling"  $g_+(\mu), g_3(\mu), g'(\mu), \delta v_+^2(\mu), \delta v_3^2(\mu)$  and  $\delta Z_{3Y}(\mu)$ . The running of  $\delta v_{+,3}^2(\mu)$  is zero since it must be proportional to a light fermion mass, which we neglect<sup>†</sup>. Therefore,

$$\delta v_{+,3}^2(m_t) = \delta v_{+,3}^2(M) \quad . \quad (8)$$

For the other runnings, one immediately obtains ( $t \equiv \log \mu^2$ ),

$$\begin{aligned} \frac{dg_+^2}{dt} &= 3 \frac{g_+^4}{(4\pi)^2} + \dots \quad , \quad \frac{dg_3^2}{dt} = \frac{10}{3} \frac{g_3^4}{(4\pi)^2} + \dots \\ \frac{dg'^2}{dt} &= \frac{50}{9} \frac{g'^4}{(4\pi)^2} + \dots \quad , \quad \frac{d}{dt} \delta Z_{3Y} = \frac{1}{6(4\pi)^2} + \dots \end{aligned} \quad (9)$$

From diagrams such as the ones in fig. 1, but with all fermions now contributing since, apart from top, all others are lighter than  $M_Z$ . Ellipses in eq. (9) stand for the contribution of the gauge bosons and the Higgs<sup>‡</sup>.

So far  $\alpha_s = 0$ . The incorporation of QCD corrections proceeds in two steps. Firstly, all couplings and masses develop a dependence on  $\mu$  through gluon loops, so that for instance  $m_t$  becomes  $m_t(\mu)$ . Secondly, the matching conditions and the RGE's (6)-(9) get corrected by an  $\alpha_s$ -dependent term. In general, the calculation of this term in the matching conditions is a hard two-loop calculation, and thinking in

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★ Hence we neglect possible terms  $\sim \log M_W/M_Z$ .

† We also neglect the contribution of the gauge bosons and the Higgs since they do not have QCD corrections. This simplifies the analysis enormously.

‡ We note again that this contribution will not have QCD corrections to the order we are working.

terms of an EFT does not help much to obtain this number. It has to be calculated. For the case of eq. (6), this was done in ref. [3] with the result,

$$\begin{aligned} \frac{v_+^2(M) - v_3^2(M)}{v_+^2(M)} &= \frac{v_+^2(m_t) - v_3^2(m_t)}{v_+^2(m_t)} = \\ &= \frac{3}{(4\pi)^2} \frac{m_t^2(m_t)}{v_+^2(m_t)} \left[ 1 - \frac{2}{9} \frac{\alpha_s(m_t)}{\pi} (\pi^2 - 9) + \mathcal{O}(\alpha_s^2) \right] \end{aligned} \quad (10)$$

where, as nicely explained in ref. [12], the scale in  $\alpha_s(\mu)$  and  $m_t(\mu)$  clearly has to be  $\sim m_t$  (and not  $M_Z$  or  $m_b$ ) because it is nothing but a matching condition at  $\mu = m_t$ . This is the  $\rho$  parameter. We shall see below that  $v_+^2(m_t) = (\sqrt{2}G_F)^{-1}$ , where  $G_F$  is the  $\mu$ -decay constant. In ref. [12] eq. (10) was written in terms of the pole mass  $m_t$  using the following relation connecting  $m_t(\mu)$  and  $m_t$ :

$$m_t = m_t(m_t) \left( 1 + \frac{4\alpha_s(m_t)}{3\pi} + 10.95 \left( \frac{\alpha_s(m_t)}{\pi} \right)^2 + \dots \right), \quad (11)$$

to  $\mathcal{O}(\alpha_s)$ . Recently<sup>[13]</sup>, Sirlin has noted that because this relation has a very large coefficient accompanying the  $\mathcal{O}(\alpha_s^2)$  contribution, one can take eq. (10) as a very interesting starting point for the analysis of the  $\mathcal{O}(\alpha_s^2)$  corrections to eq. (10)<sup>[14]</sup>. Within the EFT approach it comes out very naturally.

Corrections of  $\mathcal{O}(\alpha_s)$  to eq. (7) can be disregarded because they are subleading in our leading-log calculation. Similarly eq. (8) also remains valid even when  $\alpha_s \neq 0$ .

The inclusion of QCD corrections also modifies the RGE's (9) but this modification has been known since the times of RG applications to grand unified theories<sup>[15]</sup>. All it amounts to is multiply every quark contribution to eq. (9) by  $(1 + \alpha_s(\mu)/\pi)$ . Therefore one obtains,

$$\begin{aligned}
\frac{dg_+^2}{dt} &= \frac{g_+^4}{(4\pi)^2} \left[ 2 \left( 1 + \frac{\alpha_s(t)}{\pi} \right) + 1 \right] + \dots \\
\frac{dg_3^2}{dt} &= \frac{g_3^4}{(4\pi)^2} \left[ \frac{7}{3} \left( 1 + \frac{\alpha_s(t)}{\pi} \right) + 1 \right] + \dots \\
\frac{dg'^2}{dt} &= \frac{g'^4}{(4\pi)^2} \left[ \frac{23}{9} \left( 1 + \frac{\alpha_s(t)}{\pi} \right) + 3 \right] + \dots \\
\frac{d}{dt} \delta Z_{3Y} &= \frac{1}{6(4\pi)^2} \left( 1 + \frac{\alpha_s(t)}{\pi} \right) + \dots
\end{aligned} \tag{12}$$

to be supplemented with the running of  $\alpha_s(t)$ ,

$$\frac{d\alpha_s}{dt} = -\frac{\beta_0}{(4\pi)} \alpha_s^2 \quad , \quad \beta_0 = 11 - \frac{2}{3}n_f \quad , \quad n_f = 5 \text{ flavors} \quad . \tag{13}$$

Given that  $g_+^2, g_3^2$  and  $g'^2$  are all rather smaller than  $g_s^2 \equiv 4\pi\alpha_s$ , a reasonable approximation is to take into account the running of  $\alpha_s$  in eqs. (12) while keeping the  $g_+, g_3$  and  $g'$  frozen at a given value. This is tantamount to resumming the leading log's accompanying powers of  $\alpha_s$  but not those accompanied by powers of  $g_+, g_3$  and  $g'$ .

With all this, one can now go about computing a typical physical quantity like for instance  $\Delta r_W^*$ . In the EFT language this is obtained in the following way. According to the Lagrangian (4) the physical  $W$  and  $Z$  masses are given by the equations

$$\begin{aligned}
M_W^2 &= \frac{g_+^2(M)}{4} v_+^2(M) \\
M_Z^2 &= \frac{g_3^2(M)}{4c^2(M)} (v_3^2(M) + 4M_Z^2 \delta Z_{3Y}(M))
\end{aligned} \tag{14}$$

where  $c^2(M) = \cos^2 \theta_W(M)$  and  $\tan \theta_W(M) \equiv g'(M)/g_3(M)$ .

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\*  $\Delta r_W$  is the same as the more familiar parameter  $\Delta r$  defined by Marciano and Sirlin<sup>[16]</sup> but without the running of  $e(\mu)$ .

Following the EFT technique, at the scale of the W mass one should integrate out the W boson. This gives rise to the appearance of 4-fermion operators that mediate  $\mu$  decay, with strength  $G_F(M)/\sqrt{2}$ . The matching condition therefore becomes

$$\frac{G_F(M)}{\sqrt{2}} = \frac{g_+^2(M)}{8M_W^2} = \frac{1}{2v_+^2(M)} , \quad (15)$$

but since  $G_F(\mu)$  does not run<sup>[17]</sup> one can see that actually  $v_+^2(M) = \sqrt{2}G_F$ , where  $G_F$  is the Fermi constant as measured in  $\mu$  decay. Therefore

$$\frac{G_F}{\sqrt{2}} = \frac{g_+^2(M)}{8M_W^2} = \frac{e^2(M)}{8M_W^2} \left[ \frac{g_+^2(M)}{g_3^2(M)} \frac{1}{s^2(M)} \right] , \quad (16)$$

where  $e^2(\mu)$  is the running electromagnetic coupling constant. The quantity  $\Delta r_W$  is defined as

$$\frac{G_F}{\sqrt{2}} = \frac{e^2(M)}{8M_W^2 s^2} (1 + \Delta r_W) .$$

Consequently,

$$1 + \Delta r_W = \frac{s^2}{s^2(M)} \frac{g_+^2(M)}{g_3^2(M)} \quad (17)$$

where  $s^2 \equiv 1 - M_W^2/M_Z^2$  is Sirlin's combination<sup>[18]</sup>.

Since we are only interested in resumming  $\alpha_s$  corrections we can approximate  $\Delta r_W$  in eq. (17) by

$$\Delta r_W \approx \frac{c^2 - s^2}{s^2} \frac{g_3^2(M) - g_+^2(M)}{g^2} - \frac{c^2}{s^2} \frac{v_+^2(m_t) - v_3^2(m_t)}{v^2} + \frac{4M_Z^2}{v^2} \frac{c^2}{s^2} \delta Z_{3Y}(M) . \quad (18)$$

Integration of eqs. (12) and (13), with the boundary conditions (6)-(8), yields

$$\begin{aligned}
\frac{g_+^2(M)}{g_3^2(M)} &\approx 1 + \frac{g^2}{(4\pi)^2} \left[ -\frac{1}{3} \log \left( \frac{M^2}{m_t^2} \right) + \log \left( \frac{\alpha_s(M)}{\alpha_s(m_t)} \right)^{-\frac{4}{\beta_0}} \right] \\
\delta Z_{3Y}(M) &\approx \frac{1}{6(4\pi)^2} \left[ \log \left( \frac{M^2}{m_t^2} \right) + \log \left( \frac{\alpha_s(M)}{\alpha_s(m_t)} \right)^{-\frac{4}{\beta_0}} \right]
\end{aligned} \tag{19}$$

so that  $\Delta r_W$  is, finally,

$$\begin{aligned}
\Delta r_W &\approx -\frac{c^2}{s^2} \frac{3}{(4\pi)^2} m_t^2(m_t) G_F \sqrt{2} \left[ 1 - \frac{2}{9} \frac{\alpha_s(m_t)}{\pi} (\pi^2 - 9) \right] + \\
&+ \frac{g^2}{(4\pi)^2} \frac{1}{2} \left( \frac{c^2}{s^2} - \frac{1}{3} \right) \left[ \log \left( \frac{M^2}{m_t^2} \right) + \log \left( \frac{\alpha_s(M)}{\alpha_s(m_t)} \right)^{-\frac{4}{\beta_0}} \right]
\end{aligned} \tag{20}$$

with  $\beta_0 = 23/3$ . In the second term of eq. (20) one has actually resummed all orders in  $\alpha_s^n \log^n$ . It is here that the powerfulness of the RG and EFT has proved to be very useful. Therefore we learn that while the term proportional to  $m_t^2(m_t)$  comes from matching, and has therefore a well-defined scale  $\mu \simeq m_t$ ; the term proportional to  $g^2$  comes from running, instead, which in turn means that it has to depend on the two scales between which it is running,  $\mu \simeq M$  and  $\mu \simeq m_t$ . From the point of view of an EFT aficionado, eq. (20) is somewhat unconventional in that it considers matching conditions (the  $\alpha_s(m_t)$  term) together with running (the  $\alpha_s(M)/\alpha_s(m_t)$  term) both at one loop. From the QCD point of view the former is a next-to-leading-log term whereas the latter is a leading-log one. The reason for taking both into account is of course that the  $\alpha_s(m_t)$  term is multiplied by the  $m_t^2 G_F$  combination, which is large.

If one takes the  $\alpha_s(M)/\alpha_s(m_t)$  logarithmic term, expands it in powers of  $\alpha_s$  and uses eq. (11) to rewrite eq. (20) in terms of the pole mass, one of course reobtains eq. (1) to the given order.

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## FIGURE CAPTIONS

Fig. 1. Diagrams contributing to the matching conditions, eqs. (6)-(7).

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